

**Worksheet – Order of Magnitude Estimation**

“About how many chairs are in your classroom? Is it more like 2, or 7, or 15?”

You probably didn’t really need to count. Just looking at things, you have a feel for whether there are “a couple”, or “a few”, or “a dozen”. You have done it enough times that you now have a feel for it, an *intuition*.

“How many people have you talked with today? Is it more like 1, or 10, or 100?”

“How much time have you spent reading this? Is it more like 1 minute, or 10 minutes, or 100 minutes?”

“How far away is home? Is it more like 1 mile, or 10 miles, or 100 miles?”

“How tall is that tree? Is it more like?

- (a) one foot, or
- (b) ten feet, or
- (c) A hundred feet, or
- (d) a thousand feet, or
- (e) ten thousand feet?”

Hmm... Could it be 1 foot? Well, unless it’s a seedling, it has to be a lot bigger than 1 foot. So (a) is out. Could it be 10,000 feet? Well... Isn’t a mile only a few thousand feet? It doesn’t seem the tree could be a mile tall. So it has to be less than 10000 feet. So it’s (b), (d), or (e). Could it be 10 feet? What do we know that’s about 10 feet big? Hmm... How tall are people? So if the tree is a couple of people high, (b) sounds good. Oh, but that means (d), 1,000 feet, would be how high? 100 people high!? That doesn’t sound right. So it looks like (b) or (c), depending on how tall the tree is.

A process of making an educated guess of the magnitude of an unknown quantity, based only on the power of ten closest to the number, is called *order of magnitude estimation*. In such a process, we look only at how many zeros a quantity has. “One, ten, hundred, thousand, ten thousand, hundred thousand, million, ten million, hundred million, billion, etc...”

one	1	$10^0$
ten	10	$10^1$
hundred	100	$10^2$
thousand	1000	$10^3$
Ten thousand	10,000	$10^4$
Hundred thousand	100,000	$10^5$
million	1,000,000	$10^6$
Ten million	10,000,000	$10^7$
hundredmillion	100,000,000	$10^8$
billion	1,000,000,000	$10^9$
Ten billion	10,000,000,000	$10^{10}$
Hundred billion	100,000,000,000	$10^{11}$
trillion	1,000,000,000,000	$10^{12}$
Ten trillion	10,000,000,000,000	$10^{13}$
Hundred trillion	100,000,000,000,000	$10^{14}$

When you estimate, you are aiming to get within the right *order of magnitude*. You are not looking for a precise answer!

The goal of OOM estimation is to allow us to do calculations quickly using mental math (not fuzzy math!). Hence, we don't use a calculator when doing such estimates. Just round all factors to the nearest power of ten and then use the laws of exponents to do the calculations.

Some things to note:

- When making OOM estimates, round numbers up when the leftmost digit is 5 or above. For example, 4,302 is about  $10^3$  but 8,318 is about  $10^4$ . Of course, in extreme cases, this causes some accuracy to be lost.
- You can make an OOM estimate more accurate by using *one significant figure* times a power of ten. For example, 4,302 would be  $4 \times 10^3$  and 4,790 would be  $5 \times 10^3$ . The drawback with this is that it makes mental math a bit more difficult.

**Example:**

$$\frac{56,238 \times 110,176,334}{860,413} \approx \frac{10^5 \times 10^8}{10^6} = \frac{10^{13}}{10^6} = 10^7 :$$

What is the actual answer? How close is the estimate?

Can you do a one significant figure approximate of the above calculation?

**Homework:**

1. Approximately how many hours will an average person sleep during her life?
2. How many glasses of water would it take to fill the school swimming pool?
3. What is the weight, in pounds, of the all the students in your school?
4. How many painted turtles would it take stacked on top of each other to be as tall as the water tower?
5. How many CDs (regular, not MP3) could you listen to in the car on a trip from the east coast to the west coast?
6. The moon is roughly 240,000 miles away. Estimate how many cars it would take when placed end-to-end to reach from the earth to the moon?
7. If you watched every single episode of the Simpsons (22 seasons) back-to-back, how many hours would you be sitting in front of the TV?
8. How many hot dogs will be eaten at Tiger Stadium during a one year season?
9. Estimate the number of buckets of popcorn made at Quality 16 movie theater on a Friday night?
10. How many gallons of gasoline are used by cars each year in the United States?
11. How many minutes will be spent on the phone (talking and texting) by all the students in your school this year?
12. How many slices of pizza will be eaten in Michigan this year?