

Activity - Analysis of an Experiment

The presentation and analysis of experimental results is an essential part of physics. The tables below show the results of an experiment in which the time it takes to empty a cylindrical can was measured. Two variables were used: (1) the height of water in the can, and (2) the diameter of the drain hole. As you would expect, this time depends on the size of the hole and the amount of water in the can.

You are asked to present and analyze these results in a form which will enable you to predict the outcome of similar experiments.

To find the dependence on the *size of the hole*, four large cylindrical containers of water of the same size were emptied through relatively small circular openings of different diameters. To find the dependence on the *amount of water*, the same containers were filled to different heights. Each measurement was repeated several times, and the averages of the times (in seconds) that each container took to empty have been entered in the table.

Note: A stop watch operated by a human hand cannot be trusted to measure less than a tenth of a second. The last digit in each time entry in the table may be in error by one unit either way. Therefore, the relative (or fractional) error is larger for shorter times than for longer times. Explain?

Time vs. Height (cm) and Diameter (cm) in Seconds

Diameter (in cm)	Height (cm)			
	30.0	10.0	4.0	1.0
1.5	73.0	43.5	26.7	13.5
2.0	41.2	23.7	15.0	7.2
3.0	18.4	10.5	6.8	3.7
4.0	6.8	3.9	2.2	1.5

All the information we need is in the above tables, but a **graphical presentation** will enable us to make predictions and will greatly facilitate the discovery of mathematical relationships.

Graphing: First, plot (by hand) the Time versus the Diameter of the opening for a constant height, say 30.0 cm. It is customary to mark the *independent variable* (in this case, the diameter d) on the horizontal axis and the *dependent variable* (here the time t) on the vertical axis. To get maximum accuracy on your plot, you will want the curve to extend across the whole sheet of paper. Choose your scales on the two axes accordingly, without making them awkward to read. Connect the points by a smooth curve (called a "best fit").

Now graph the *time vs. diameter* data for other three heights on the same set of axes. Use colored pencils to distinguish between graphs.

Next, graph (by hand) *the Time vs. Height* (for constant diameter) from the values in table 2. Draw all four graphs on the same set of axes. Make a best fit curve/line through your data points.

Using a Spreadsheet: Now go to the computer lab to draw your graphs with a spreadsheet program (Microsoft Excel). Open a new workbook, save it with an appropriate name, and type in the data table into sheet 1 of the spreadsheet (double clicking on the tab at the bottom of the screen labeled “sheet1” will allow you to change it’s name to “Data”.)

After entering the data, format your data table with borders and centering, etc... Then select the diameter and time data by holding down the mouse button and dragging to the right and down in table. Click the “Chart Wizard”  in the toolbar. Follow the steps to make your graph. Be sure to read the instructions given by the chart wizard (you will need to tell the Chart Wizard if your data is in Columns). When you are on the last step of the Chart Wizard, choose “set up chart in new sheet” as opposed to “make chart an object in current sheet” before hitting “finish.”

Repeat this process for the Time vs. Height data by selecting the heights and times (but not the diameters). This time be sure to indicate that your data is in “Rows.”

Use Microsoft Excel to draw a *best fit curve* (trend line) for all of your plots. Provide an “equation of best fit” for the top plot on each graph. To do this, click once on one data set and select “Add Trend line” from the “Chart” menu. Do this for all your data sets. You can double click on the trend lines to change their colors or line type so they show up better on your final printout. When you are finished you should have a computer generated graph that looks similar to your hand-written graph of Time vs. Diameter. Although you can use the curves to *interpolate* between your measurements and roughly *extrapolate* beyond them, you have not yet found an algebraic expression for the relationship between T and D, and T and H. To do this, click twice on one of the trend lines and choose “Options” then “Show Equation on Graph.”

Making the Graph Linear: From your graph you can see that t decreases rather rapidly as d increases; this suggests some type of *inverse relationship*. Furthermore, you may argue that the time of flow should be simply related to the ‘area’ of the opening, since the larger the area of the opening, the more water will flow through it in the same time. Try a plot of t versus $1/d^2$ instead of time vs. diameter.

To do this, add a column for the values of $1/d^2$ in your table 1 and, again choosing a convenient scale, plot t versus $1/d^2$ and connect the points with a smooth curve. Find the algebraic relationship between t and $1/d^2$ by making a trend line and displaying an equation?

Extrapolation: Now investigate the dependence of t on h when the diameter of the opening stays fixed. Take the case of $d = 1.5$ cm, which is the first row. Make a plot in which h will be marked on the horizontal axis and connect your points by a curve. Extrapolate the curve toward the origin

Unlike the relationship between t and d , there is no simple geometric consideration to guide us to the right mathematical relation between t and h . Why? You can try to guess it from the curve (or use your spreadsheet equation). It may be helpful to rotate the graph paper through 90° and look first at h as a function of t , and then at t as a function of h . If you succeed, check by proper graphing to see if the same kind of relation between t and h holds for $d = 5.0\text{cm}$. (Q8) Does it?

Questions:

- 1.) What type of relationship exists between Time and Diameter?
- 2.) Are your graphs of Time vs. Diameter the same basic shape for all four heights? Would you expect this?
- 3.) Do your graphs of Time v. Diameter pass through the origin? Would you expect them to?
- 4.) Use your graph of Time v. Diameter (and the equation you generated) to predict the time it would take for the following:
 - a.) a can filled with 30cm of water and a diameter of 2.5cm to drain.
 - b.) a can filled with 30cm of water and a diameter of 5cm to drain.
- 5.) What type of relationship exists between Time and Height?
- 6.) Are your graphs of Time vs. Height the same basic shape for all four diameters? Would you expect this?
- 7.) Do your graphs of Time v. Height pass through the origin? Would you expect them to?
- 8.) Invent a way that you could use your graphs to determine the time to drain if the hole diameter was 1.3 cm and the height of water was 8.0 cm? Make your best guess.
- 9.) What type of relationship is there between Time and $1/d^2$?
- 10.) Use your graphs (and the equations you generated) to predict the time it would take for the following:
 - a.) a can with a hole diameter of 1.5cm that is filled with 15.0cm of water.
 - b.) a can with a hole diameter of 2.5cm that is filled with 10.0cm of water.

Extension:

Use MS Excel™ and the “AVERAGE” function to draw a graph of time vs. height when the diameter is 3.5cm. Display a trend line with an equation and use that equation to answer #8 above.